Shock-Induced Anisotropy in Ferromagnetic Material. I. Domain-Theory Analysis of Single-Crystal Behavior*

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Shock-induced demagnetization produced by strain-induced anisotropy is considered in cubic single-crystal ferromagnetic material. Equilibrium thermodynamics, along with established methods of ferromagnetic domain theory, are used to predict energy expresssions, magnetization curves, and domain structure in the magnetic material behind the shock wave. In particular, specific expressions for the exchange energy and magnetic self-energy are obtained. They are predicted to increase as the fourth root of the strain and are small compared to the induced anisotropy energy in the region of large elastic and plastic strain. Calculations are made for yttrium iron garnet.

I. INTRODUCTION

When magnetic material is subject to a strong shock wave and at the same time biased by an external magnetic field applied parallel to the shock front, a reduction in magnetization is observed. By this method shock waves are used to study the magnetic behavior of materials subject to extreme states of stress. In ferromagnetic material three mechanisms have been identified as contributing to shock-induced demagnetization. These are first-order crystallographic phase transitions in which total demagnetization is observed to occur,[1,2] second-order phase transitions between ordered and disordered magnetic states,[3,4] and shock-induced anisotropy in which nonhydrostatic strains along with magnetoelastic properties of the material produce deviations from magnetic saturation.[2,5-7] This paper is concerned with the last.

The problem of shock-induced anisotropy is best understood by considering the model used to describe it. Referring to Fig. 1, consider an infinite half-space of ferromagnetic material contained in the region $x>0$. Planar impact at the interface $x=0$ creates a plane shock wave S propagating in the positive x direction. This creates in the region behind the shock wave an infinite slab of ferromagnetic material subject to a state of uniform uniaxial strain.[8] During and following shock initiation, the ferromagnetic material is subject to a transverse magnetic field H_e sufficient to induce magnetic saturation in the material in front of the propagating shock wave. Behind the shock wave a reduction in magnetization occurs. This is the observed shock-induced demagnetization and is, in the present work, a consequence of the magnetoelastic properties of the material.

Theoretical consideration of shock demagnetization produced by shock-induced anisotropy parallels methods used in predicting other ferromagnetic behavior. This consists of writing an energy expression sufficient to describe the magnetic and mechanical properties of the ferromagnetic slab behind the shock wave and minimizing this energy with respect to some parameter which determines the magnitude of transverse magnetization in the slab. By this method the equilibrium magnetization and, hence, shock demagnetization, is determined. These general statements will be more fully described in Secs. II–VI. Early work on the shock-induced anisotropy effect considered a total energy consisting of the interaction energy $E_H=-\mathbf{H}_e\cdot\mathbf{M}_s$ along with the magnetoelastic energy E_{me} and the crystal anisotropy energy E_k.[2,5,7] This is sufficient to predict shock-induced demagnetization. These early papers represent a significant contribution to the understanding of shock-induced anisotropy. They, however, ignored energy terms which are known to significantly contribute to ferromagnetic behavior, viz., the exchange energy and the demagnetizing energy. These energy terms were considered in a micromagnetic theoretical treatment of shock-induced anisotropy.[9] However, micromagnetic theory is, at present, mathematically cumbersome and limited in application. The gap between earlier work and the sophisticated methods of micromagnetic theory is spanned by the ferromagnetic domain theory. Its concepts are well developed in the literature.[10] The primary objective of the present work is to apply the established methods of ferromagnetic domain theory to the problem of shock demagnetization produced by shock-induced anisotropy. Specific objectives are as follows: The magnetic domain structure expected to nucleate after shock passage will be predicted. A total magnetic energy expression will be obtained. In particular, explicit expressions for the exchange and demagnetization energy will be determined. The error resulting from ignoring these terms, as has been done in previous work, will also be determined. Magnetization curves for the material behind the shock wave will be obtained and conditions necessary for shock demagnetization to occur ascertained.

The present work will be restricted to the shock-induced anisotropy effect in cubic single-crystal ferromagnetic material. This is preparatory to understanding the similar effect in polycrystalline material, which is the topic of the following paper.[11] The region of strain considered in this work will be in the elastic range but at strains which are a sizable fraction of the Hugoniot elastic limit of the material. This is consistent with the order of strain obtained in earlier magnetic shock work.[2,5,6] Extension to the plastic region requires an additional assumption.[12]

This article is directed to workers in the field of shock-wave physics or people interested in the magnetic re-

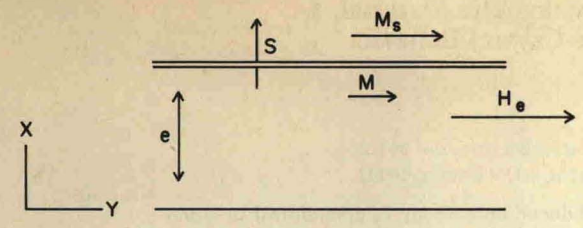

FIG. 1. Geometry for shock-induced demagnetization. Infinite half-space of ferromagnetic material in region $x > 0$. Uniaxial strain in region behind the shock S. Applied field H_e in transverse direction. Reduction in magnetization occurs across the shock front.

sponse of material to dynamic loading. To the specialist in magnetics, for whom the methods of ferromagnetic domain theory are well known, some of the concepts will be self-evident.

Presentation of this article is in the following order: The various energy terms required in the analysis are defined in Sec. II. This section also includes a discussion of the induced anisotropy effect. A common misconception has held that within the validity of conventional first-order magnetoelastic theory, ignoring crystal anisotropy energy, the axis of uniaxial strain defines an easy or hard direction of magnetization. This is not true for arbitrary crystal axis orientation with respect to the axis of uniaxial strain. An interesting consequence of this is that total shock-induced demagnetization is not expected, regardless of the magnitude of strain. In Sec. III, a ferromagnetic domain-theory analysis is presented for the shock-induced anisotropy effect. In Sec. IV, magnetization curves and conditions for shock demagnetization are determined. The results are applied to yttrium iron garnet in Sec. V and discussed in terms of relative contributions of the various magnetic energy expressions to the shock-induced anisotropy effect. Experimental shock demagnetization results on polycrystalline YIG are presented in the following article.[11]

II. THERMODYNAMIC ENERGY

Thermodynamics of a rigid ferromagnet are used to describe the shocked material.[13] A rigid ferromagnet is a thermodynamic system for which the functional dependence of the energy $E(S, M_i, e_{ij})$ is reduced to the dependence $E(S, M_i)$ by maintaining the state of strain e_{ij} constant. M_i is the magnetization and S is the entropy. A rigid ferromagnet implies that each lattice point is stationary and is not subject to motion by the forces present. The strain e_{ij} is maintained constant by the inertia of the material after passage of the shock wave and will remain so until relieved by perturbing waves (a problem only when finite boundaries are considered).[14]

A total thermodynamic energy expression sufficient for a phenomenological description of a rigid anisotropic ferromagnet is given by

$$E = E_H + E_d + E_{ex} + E_K + E_{me}. \tag{1}$$

Each term will be identified briefly. Complete developments can be found in many excellent treatments in the literature.[10,13,15,16] The elastic energy is neglected in this expression. A function of strain, only it will be constant in a rigid ferromagnet.

The first terms is

$$E_H = -\mathbf{H}_e \cdot \mathbf{M}_s. \tag{2}$$

This is the interaction energy of the magnetic material in the external applied field $\mathbf{H}_e$.

The second term is

$$E_d = -\tfrac{1}{2}\mathbf{H}_d \cdot \mathbf{M}_s. \tag{3}$$

This is the self-energy or demagnetizing energy of the magnetic system. $\mathbf{H}_d$ is the demagnetizing field and originates from magnetic surface and volume poles. This energy is intrinsically positive. Domain structure in ferromagnetic material occurs in an attempt to reduce the demagnetizing energy.

The third term is the ferromagnetic exchange energy. The magnetization gradients are found to be adequate thermodynamic variables for a phenomenological description of this energy. A quadratic form

$$E_{ex} = A_{ij} \frac{\partial \alpha_K}{\partial x_i} \frac{\partial \alpha_K}{\partial x_j}$$

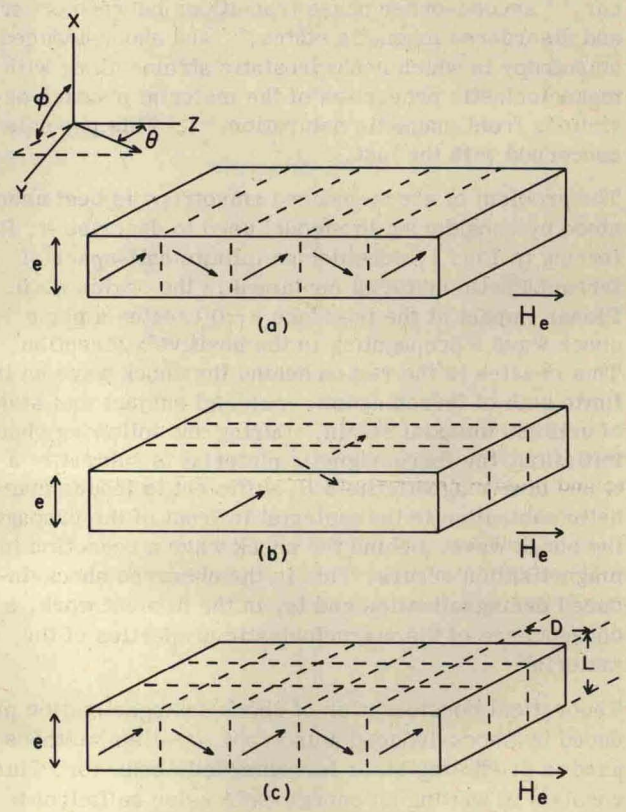

FIG. 2. (a) Model for platelike domain structure perpendicular to the applied field. (b) Model for platelike domain structure parallel to the applied field. (c) Model for needle-shaped domain structure oriented along axis of uniaxial strain. Polar angles define direction of magnetization during transition through domain wall.

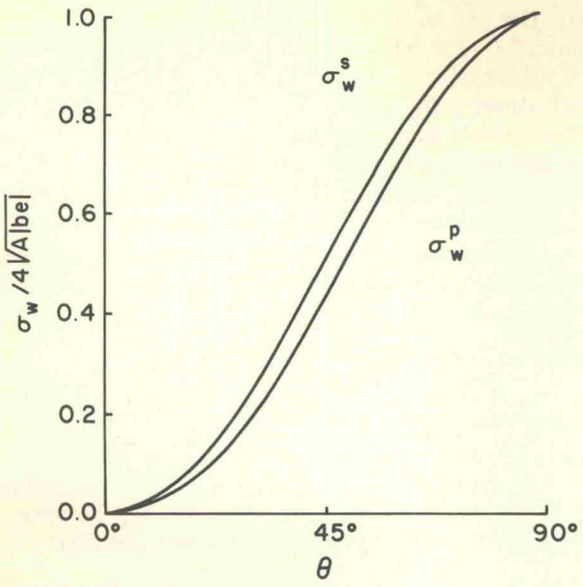

FIG. 3. Domain-wall energy as a function of θ. σ_w^s corresponds to walls shown in Fig. 2(a). σ_w^p corresponds to walls shown in Fig. 2(b).

is usually sufficient. The α_K's are the direction cosines of the magnetization vector. That is, $M_K = M_s\alpha_K$, where M_s is the saturation magnetization of the ferromagnetic material. For cubic symmetry, this expression reduces to

$$E_{ex} = A[(\nabla\alpha_1)^2 + (\nabla\alpha_2)^2 + (\nabla\alpha_3)^2], \tag{4}$$

where A is the exchange constant.

The fourth term is the crystalline anisotropy energy. From conventional magnetoelastic theory, it is given by

$$E_K = K_{ijkl}\alpha_i\alpha_j\alpha_k\alpha_l.$$

For cubic symmetry, it becomes

$$E_K = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2). \tag{5}$$

In this paper, interest lies in the shock-induced anisotropy. In shock-wave studies, strains in the large elastic and plastic regions are obtained.[12] For many magnetic materials, the crystalline anisotropy energy is 10—30 times smaller than the induced anisotropy energy in this strain region. For this reason the crystalline anisotropy will be ignored.

The last term is the magnetoelastic energy. From conventional magnetoelastic theory, it is given by

$$E_{me} = b_{ijkl}e_{ij}\alpha_k\alpha_l, \tag{6}$$

where b_{ijkl} is the fourth-rank magnetoelastic tensor. For cubic symmetry, this becomes

$$E_{me} = b_1(\alpha_1^2e_{11} + \alpha_2^2e_{22} + \alpha_3^2e_{33}) + 2b_2(\alpha_1\alpha_2e_{12} + \alpha_2\alpha_3e_{23} + \alpha_3\alpha_1e_{31}). \tag{7}$$

The magnetoelastic energy is of primary interest in the shock-induced anisotropy effect.

For a single-crystal slab of ferromagnetic material with crystal axis arbitrarily oriented with respect to the axis of uniaxial strain, the strain tensor can be written

$$e_{ij} = en_in_j, \tag{8}$$

where n_j is a component of a unit vector directed along the axis of uniaxial strain. $e = (\rho_0/\rho) - 1$ is the strain along this axis, where ρ_0 and ρ are the initial and final densities, respectively. The magnetoelastic energy [Eq. (6)] becomes

$$E_{me} = eb_{ijkl}n_in_j\alpha_k\alpha_l. \tag{9}$$

This can be written

$$E_{me} = C_{kl}\alpha_k\alpha_l, \tag{10}$$

where

$$C_{kl} = eb_{ijkl}n_in_j. \tag{11}$$

This manipulation is very convenient since it allows the familiar techniques developed for analyzing symmetric second-rank tensors to be used in analyzing the fourth-rank magnetoelastic tensor for a given state of uniaxial strain. For cubic symmetry the matrix array representing the second rank tensor in Eq. (11) becomes

$$[C_{kl}] = e\begin{pmatrix} b_1n_1^2 & b_2n_1n_2 & b_2n_1n_3 \\ b_2n_1n_2 & b_1n_2^2 & b_2n_2n_3 \\ b_2n_1n_2 & b_2n_2n_3 & b_1n_3^2 \end{pmatrix}. \tag{12}$$

The principal axes of the representation quadric for this second-rank symmetric tensor give the easy and hard directions of magnetization produced by the induced uniaxial strain. The eigenvalues are the magnetoelastic energies when the magnetization vector lies along the corresponding principal axes.[17] It should be noted that the principal axes depend only on the direction of the axis of uniaxial strain with respect to the crystal axes and not on the magnitude of strain since the eigenvectors will be functions of the n_i and independent of e. Of more interest is the fact that the axis of uniaxial strain will not, in general, coincide with a principal axis and hence will not define an easy or hard direction of magnetization. There are special cases, such as uniaxial strain along a ⟨100⟩ or a ⟨111⟩ axis, in which the strain axis and a principal axis coincide. This has the following implication: First-order conventional magnetoelastic theory[16] predicts that in any finite magnetic field, strain-induced anisotropy cannot produce

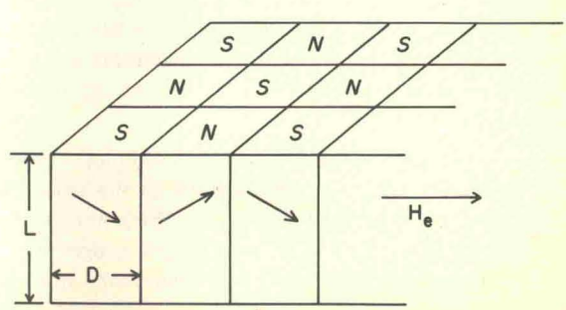

FIG. 4. Surface pole distribution for magnetostatic potential problem. There is pole distribution on both upper and lower surfaces.

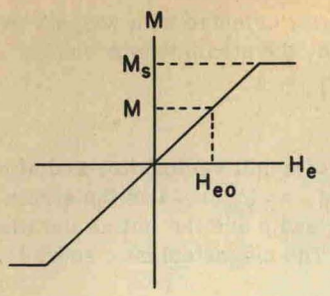

FIG. 5. Predicted magnetization curve. The magnetization M determines the extent of the shock-induced demagnetization that would occur for a specimen initially saturated in a field H_{e0}.

total shock demagnetization except in the special cases stated previously. This can be seen in the following way: The geometry of the shock demagnetization problem (Fig. 1) defines an axis of uniaxial strain with a perpendicular applied field. The single-crystal axes may be arbitrarily oriented. The direction of easy magnetization will not, in general, coincide with the axis of uniaxial strain. In the limit of vanishingly small applied field H_e, the magnetization will lie along this easy axis. Its direction along this axis will be such that $-\mathbf{H}_e \cdot \mathbf{M}_s$ is minimal. This will give a nonzero component of $\mathbf{M}$ in the direction of H_e. In a polycrystalline material all orientations of crystallites occur. Each will contribute to the transverse magnetization. This may explain, at least in part, why shock-induced demagnetization observed by Shaner and Royce[5] in YIG was less than expected.

III. DOMAIN-THEORY ANALYSIS

In the domain-theory analysis of shock-induced anisotropy, two single-crystal problems will be treated concurrently. These will be called the ⟨100⟩ problem and the ⟨111⟩ problem. The ⟨100⟩ problem corresponds to a state of uniaxial strain along a ⟨100⟩ axis with a perpendicular applied field. The ⟨111⟩ problem corresponds to a state of uniaxial strain along a ⟨111⟩ axis with a perpendicular applied field. These two problems have been chosen for the following reasons: In single-crystal magnetostriction, inverse of the effect considered in the present work, results are interpreted in terms of λ_{100} and λ_{111}. These magnetostriction constants represent total strain when a crystal is magnetized from the demagnetized state to saturation along the ⟨100⟩ and ⟨111⟩ axes. The problems considered in the present work are the complementary analogs of these inverse magnetostriction problems. The results clearly exhibit characteristic behavior of the shock-induced anisotropy effect. Also these results will be used in determining polycrystalline magnetic behavior.

There is an inherent weakness in ferromagnetic domain theory. A basic postulate of the theory is the existence of domain walls. However, the theory does not provide a means for determining unambiguously the domain structure for a given problem. The procedure is to assume possible domain structures consistent with other requirements of the problem and select from these, by energy considerations, the most likely domain structure. In Fig. 2, models for domain structures consistent with requirements of the present problem are shown. Domain walls normal to the strain axis are not expected. This is because the variation in the magnetization direction through the domain wall cannot be made without allowing $\nabla \cdot \mathbf{M}$ to deviate from zero. $\nabla \cdot \mathbf{M} \neq 0$ in the domain wall implies magnetic volume poles in the wall which would contribute excessively to the demagnetizing energy. This would be energetically unfavorable. That $\nabla \cdot \mathbf{M} = 0$ through the domain wall is a postulate of ferromagnetic domain theory. Also, domains of closure are not expected due to the very large induced anisotropy energy.

A. Induced Anisotropy Energy

The induced anisotropy energies for the ⟨100⟩ problem and the ⟨111⟩ problem will be obtained in this section. The energy will be obtained for the region within domains and within the walls through which the transition between adjacent domains is made. This will be done for walls of the form shown in Figs. 2(a) and 2(b).

Consider first the ⟨100⟩ problem and the domain geometry in Fig. 2(a). Transform Eq. (7) to polar coordinates using Eq. (8),

$$\alpha_1 = \sin\theta\cos\Phi, \quad \alpha_2 = \sin\theta\sin\Phi, \quad \text{and} \quad \alpha_3 = \cos\theta. \tag{13}$$

The induced anisotropy energy in a domain is easily obtained:

$$E_{me}^{\langle 100\rangle}(\text{domain}) = b_1 e \sin^2\theta. \tag{14}$$

To obtain the induced anisotropy energy in the wall the variation in $\mathbf{M}$ through the wall must be considered. The requirement that $\nabla \cdot \mathbf{M} = 0$ through the wall is equivalent to demanding that θ be constant through the wall. This requires the transition between adjacent domains to proceed by a rotation of Φ from 0 to π. The energy in the wall is

$$E_{me}^{\langle 100\rangle}(\text{wall}) = b_1 e \sin^2\theta\cos^2\Phi. \tag{15}$$

In determining Eqs. (14) and (15) from the geometry in Fig. 2(a), it should be pointed out that within a domain $\mathbf{M}$ lies in the xz plane and, therefore, $M_y = 0$ or $\Phi = 0$; while in the wall, $\nabla \cdot \mathbf{M} = 0$ implies $\partial M_z / \partial z = 0$ and, therefore, $M_y \neq 0$ which implies $\Phi \neq 0$, i.e., $\mathbf{M}$ rotates out of the xz plane in keeping $\nabla \cdot \mathbf{M} = 0$.

A slightly more difficult analysis gives for the ⟨111⟩ problem

$$E_{me}^{\langle 111\rangle}(\text{domain}) = b_2 e \sin^2\theta \tag{16}$$

and

$$E_{me}^{\langle 111\rangle}(\text{wall}) = b_2 e \sin^2\theta\cos^2\Phi. \tag{17}$$

This is most easily accomplished by subjecting the energy expression in Eq. (7) to a coordinate transformation such that the new x axis lies along the old ⟨111⟩ direction. Since the forms of the energies are the same for the ⟨100⟩ problem and the ⟨111⟩ problem we will write

$$E_{me}(\text{domain}) = be \sin^2\theta \tag{18}$$

and

$$E_{me}(\text{wall}) = be \sin^2\theta\cos^2\Phi, \tag{19}$$

where $b = b_1$ or b_2 for the $\langle 100 \rangle$ problem or the $\langle 111 \rangle$ problem, respectively.

Consider the domain configuration in Fig. 2(b). Again the energy in the domain is

$$E_{me}(\text{domain}) = be \sin^2\theta .$$

The transition through the wall proceeds in the (xz) plane by varying continuously from $-\theta$ to θ. The energy in the wall is

$$E_{me}(\text{wall}) = be \sin^2\xi, \quad -\theta \le \xi \le \theta. \tag{20}$$

Equations (18)—(20) are the primary equations derived in this section.

B. Exchange Energy

Within the concepts of ferromagnetic domain theory, the exchange energy is believed to reside only in the domain walls or transition regions between adjacent domains. The method for obtaining this domain-wall energy is through a Landau—Lifshitz domain-wall calculation.[18] This has been fully developed in the literature[10,15] and will be described only briefly here. The method consists of writing a one-dimensional integral expression for the energy in the transition region between domains. The terms which contribute to the domain-wall energy are the exchange energy [Eq. (4)] and the excess crystalline or magnetoelastic anisotropy energy incurred by the transition through the wall. It is assumed that $\nabla \cdot \mathbf{M} = 0$ ($\theta = \text{const}$) holds through the wall. This one-dimensional integral energy expression is minimized by variational calculus. The result predicts that for all points within the wall the exchange energy is equal to the excess anisotropy energy. It is found that the domain-wall energy per unit area is given by[15]

$$\sigma_w = 2\sqrt{A} \sin\theta \int_{\Phi_1}^{\Phi_2} \left| [E_{me}(\text{domain}) - E_{me}(\text{wall})]^{1/2} \right| d\Phi . \tag{21}$$

The crystal anisotropy energy has not been considered. A is again the exchange constant and Φ_1 and Φ_2 are the azimuthal orientation of the magnetization in the adjacent domains separated by the wall.

Energies for domain walls oriented as shown in Figs. 2(a) and 2(b) will be called σ_w^s and σ_w^p, respectively. For Fig. 2(a), using Eqs. (18) and (19) with Eq. (21), gives

$$\sigma_w^s = 2(A|be|)^{1/2} \sin^2\theta \int_0^{\pi} \sin\Phi \, d\Phi$$

or

$$\sigma_w^s = 4(A|be|)^{1/2} \sin^2\theta . \tag{22}$$

For Fig. 2(b), using Eqs. (18) and (20) with Eq. (21), gives

$$\sigma_w^p = 2(A|be|)^{1/2} \int_{-\theta}^{\theta} (\sin^2\theta - \sin^2\xi)^{1/2} \, d\xi .$$

Making the substitution

$$\sin\xi = \sin\theta \sin x = a \sin x$$

and using the identity

$$\cos^2 x = (1 - a^{-2}) + a^{-2}(1 - a^2 \sin^2 x),$$

one obtains

$$\sigma_w^p = 4(A|be|)^{1/2} \left((a^2 - 1) \int_0^{\pi/2} \frac{dx}{(1 - a^2 \sin^2 x)^{1/2}} + \int_0^{\pi/2} (1 - a^2 \sin^2 x)^{1/2} \right) dx .$$

This is

$$\sigma_w^p = 4(A|be|)^{1/2} [(a^2 - 1)K(a, \tfrac{1}{2}\pi) + E(a, \tfrac{1}{2}\pi)], \tag{23}$$

where K and E are complete elliptic integrals of the first and second kind and $a = \sin\theta$.

σ_w^s and σ_w^p are compared in Fig. 3. It is seen that the domain model considered in Fig. 2(b) yields a slightly lower energy. In actual crystalline material imperfections such as dislocation, impurities, etc., can significantly alter the domain-wall energy. For this reason, it is believed that the slight energy difference is not substantial enough to favor the domain structure in Fig. 2(b) over that in Fig. 2(a). Alternatively, a Boltzmann distribution predicts, at normal temperatures, roughly an even distribution of domain walls of both orientations. From this, one may conclude that ferromagnetic domain theory suggests a needle- or sliver-shaped domain structure oriented along the axis of uniaxial strain will nucleate behind the shock front. A model for this structure is shown in Fig. 2(c).

Due to the much simpler form of Eq. (22), the approximation

$$\sigma_w^p \simeq \sigma_w^s = \sigma_w = 4(A|be|)^{1/2} \sin^2\theta$$

will be made. An expression for the effective exchange energy density in Eq. (1) can be obtained by dividing

$$\sigma_w^p + \sigma_w^s \simeq 2\sigma_w$$

by the domain dimension D shown in Fig. 2(c). This gives

$$E_{ex} = 2\sigma_w / D$$

or

$$E_{ex} = [8(A|be|)^{1/2}/D] \sin^2\theta . \tag{24}$$

C. Demagnetizing Energy

The demagnetizing energy can be obtained by solving the magnetostatic boundary value problem for the magnetic surface pole distribution on two surfaces separated a distance L as indicated in Fig. 4. The solution requires only a slight variation on a problem already solved by Kittel.[19] The result is

$$E_d = 1.1(DM_s^2/L) \sin^2\theta . \tag{25}$$

L is the slab thickness, D is the domain dimension, and M_s is the saturation magnetization.

D. Total Energy

From the results of this section, Eq. (1) for the total thermodynamic energy can now be explicitly written

$$E(D, \theta) = -M_s H_e \cos\theta + be \sin^2\theta + 1.1(DM_s^2/L) \sin^2\theta + [8(A|be|)^{1/2}/D] \sin^2\theta , \tag{26}$$

where $\cos\theta$ is the component of the magnetization in the direction of the applied field.

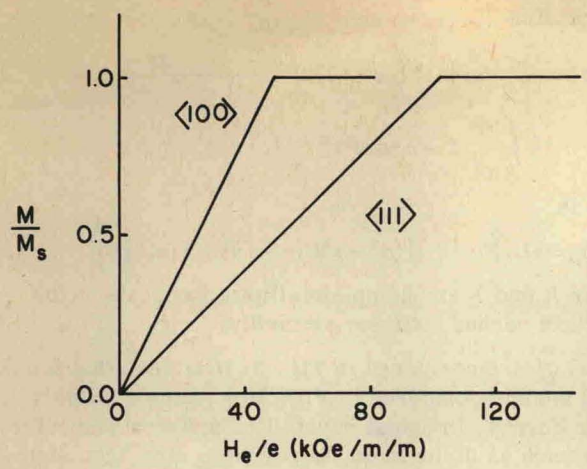

FIG. 6. Magnetization curves for the ⟨100⟩ and the ⟨111⟩ problem in YIG.

IV. EQUILIBRIUM MAGNETIC BEHAVIOR

Equilibrium thermodynamics requires that the energy expression $E(D,\theta)$ be a minimum with respect to a variation of the internal coordinates D and θ. Consider the domain-width parameter first. Minimizing with respect to D gives

$$\frac{\partial E}{\partial D}=1.1\frac{M_s^2}{L}\sin^2\theta-\frac{8(A|be|)^{1/2}}{D^2}\sin^2\theta=0.$$

This yields an expression for the domain width:

$$D=[8L(A|be|)^{1/2}/1.1M_s^2]^{1/2}. \tag{27}$$

This can be substituted back into Eq. (26) giving

$$E(\theta)=-M_sH_e\cos\theta+be\sin^2\theta+2[8.8M_s^2(A|be|)^{1/2}/L]^{1/2}\sin^2\theta \tag{28}$$

or

$$E(\theta)=-M_sH_e\cos\theta+be\sin^2\theta+\gamma|e|^{1/4}\sin^2\theta,$$

where

$$\gamma=2[8.8M_s^2(A|b|)^{1/2}/L]^{1/2}.$$

The last term in Eq. (28) will be called the equilibrium exchange and demagnetizing energy. Note that it increases as the fourth root of the strain.

From Eq. (28) the magnetization curve can be obtained. The requirement of equilibrium yields two solutions:

$$\sin\theta=0 \tag{29}$$

or

$$2(be+\gamma|e|^{1/4})\cos\theta+H_eM_s=0. \tag{30}$$

The correct solution for a given applied field is determined from the requirement that the equilibrium point be a minimum. Two cases occur which are determined by the sign of $\beta\equiv(be+\gamma|e|^{1/4})$. The sign depends on the strain e (usually negative in shock-wave experiments) and the magnetoelastic constant b. γ is always positive. For the first case ($\beta<0$), the magnetization is

$$\begin{aligned} M/M_s&=1, && H_e>-2\beta/M_s\\ &=-(M_s/2\beta)H_e, && H_e<-2\beta/M_s. \end{aligned} \tag{31}$$

The form of the magnetization curve is shown in Fig. 5. Shock-induced demagnetization is expected for $\beta<0$. For example, referring to Fig. 5, a material initially in magnetic saturation in a transverse field H_{e0} would suffer a reduction in magnetization to a value M after passage of the shock wave. For the second case ($\beta>0$), the axis of uniaxial strain defines a hard direction of magnetization. All perpendicular axes are equivalent easy directions and the magnetization curve predicted by this simple treatment will be a discontinuous jump of $2M_s$ on reversal of the applied field. This case does not, however, lead to shock demagnetization and, therefore, is not of interest in the present context.

V. DISCUSSION

Results of Sec. IV will be considered using the material properties of yttrium iron garnet. This ferrimagnetic ceramic has received attention in previous shock-induced anisotropy work because of the convenient magnitudes of its material properties. A rough value for the exchange constant of YIG obtained from molecular field theory is $A\approx3\times10^{-7}$ erg/cm. At a strain of -0.01 in YIG, which corresponds to about 25-kbar shock pressure, the predicted domain width from Eq. (27) is 20 μ. This is in agreement with other work.[7]

The equilibrium exchange and demagnetizing energy [last term in Eq. (28)] is observed to increase as the fourth root of the strain while the induced anisotropy energy increases linearly with the strain. This implies that the equilibrium exchange and demagnetizing energy would assume decreasing importance with increasing strain. For a strain of -0.01, the equilibrium exchange and demagnetizing energy is about 2% of the induced anisotropy energy. This justifies the approximation of ignoring this energy term in predicting magnetic behavior in the region of large strain as has been done in previous work.[2,5,7] It is worth noting that this approximation does not extend to all materials. In iron this neglected term represents a significant part of the energy even up to the elastic limit of the material.

Magnetization curves for the ⟨100⟩ problem and the ⟨111⟩ in YIG are shown in Fig. 6. They will be referred to in the following article.[11] The equilibrium exchange and demagnetizing energy has been ignored. The curves are plotted against the parameter H_e/e. Magnetization curves for any strain are expected to be self-similar against this parameter.

VI. SUMMARY

(i) The fourth-rank magnetoelastic tensor for a given state of uniaxial strain can be analyzed with the familiar techniques available for second-rank symmetric tensors. It was found that the axis of uniaxial strain defined an easy or hard direction of magnetization only in special cases of particular crystal orientations such as uniaxial strain along the ⟨100⟩ or ⟨111⟩ directions or in the case of magnetoelastic isotropy.

(ii) The established methods of ferromagnetic domain

theory are consistently applied to the problem of shock-induced anisotropy in ferromagnetic material. ⟨100⟩- and ⟨111⟩-oriented single crystals are considered. Using a Landau-Lifshitz domain-wall calculation, wall energies for wall normals parallel and perpendicular to the applied field are obtained. The wall energies are found to be approximately the same, the latter being slightly lower. From this, it is concluded that a needle- or sliver-shaped domain structure oriented along the axis of uniaxial strain is most likely to nucleate behind the shock front.

(iii) A total magnetic energy expression is determined for the ⟨100⟩ and the ⟨111⟩ problems. Assuming the validity of equilibrium thermodynamics behind the shock front, expressions for the domain size and the magnetization curves are obtained.

(iv) The equilibrium exchange and demagnetizing energy is found to increase as the fourth root of the strain while the induced anisotropy energy increases linearly with the strain. Thus, the former term assumes decreasing importance with increasing shock strength. It is found to be negligible in the region of large strain in yttrium iron garnet, and previous treatments[2,5,7] of the shock-induced anisotropy effect, in which the exchange and demagnetizing energy was ignored, are justified in this assumption.

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